

Limits & asymptotes

$$\lim_{x \rightarrow c} f(x) = L \quad \text{as } x \text{ gets close to } c$$

both left & right
are equal

$f(x)$ gets close to L

one-sided limits

right
hand
limit

$$\lim_{x \rightarrow c^+} f(x) = L$$

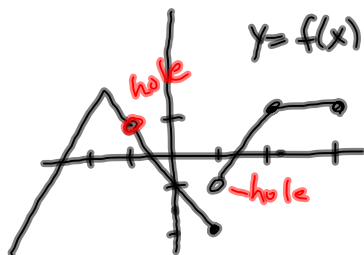
x approaches c from
the right

left
hand
limit

$$\lim_{x \rightarrow c^-} f(x) = L$$

x approaches c from
the left

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$$\lim_{x \rightarrow 1^-} f(x) = 1 \quad \lim_{x \rightarrow 1^+} f(x) = 1 \quad \lim_{x \rightarrow 1} f(x) = 1$$

↑
y coord of
hole

$$\lim_{x \rightarrow 1^-} f(x) = -2$$

$$\lim_{x \rightarrow 1^+} f(x) = -1$$

$$\lim_{x \rightarrow 1} f(x) = \text{DNE}$$

because
 $\text{lhs} \neq \text{rhs}$

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other nonexistant limits

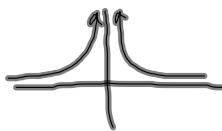
$$\lim_{x \rightarrow 0^+} \ln x = -\infty$$

dne

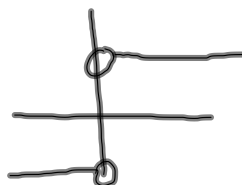


$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$$

dne



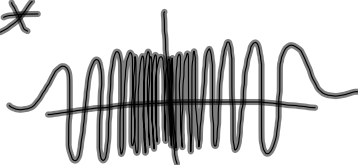
$$\lim_{x \rightarrow 0} \frac{|x|}{x} = *$$



$$\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right) = \text{dne}$$

*

by oscillation



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$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 5x + 6} = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{(x-3)(x-2)} = 6$$

hole at (3, 6)

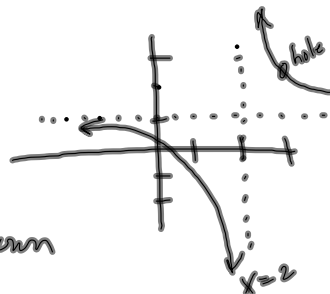
$$\lim_{x \rightarrow 2} \frac{x^2 - 9}{x^2 - 5x + 6} = \pm \infty$$

vertical asymptote at $x = 2$

$$\lim_{x \rightarrow 2^+} \frac{x^2 - 9}{x^2 - 5x + 6} = \infty \text{ up}$$

$$\lim_{x \rightarrow 2^-} \frac{x^2 - 9}{x^2 - 5x + 6} = -\infty \text{ down}$$

pos
neg



horg asymptote

$$\lim_{x \rightarrow \pm \infty} \frac{x^2 - 9}{x^2 - 5x + 6} = 1$$

H.A. at $y = 1$

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vertical asymptote

$$\lim_{x \rightarrow a} f(x) = \pm \infty \quad \text{v.A. at } x=a$$

horizontal asymptote

$$\lim_{x \rightarrow \pm \infty} f(x) = b \quad \text{HA. at } y=b$$

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